The principals of statistical analysis

(Statistikk i helsefagleg forsking)

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Example of a statistical test I

- Problem: To what extent will a congenital heart malformation (non-cyanotic) influence the motor development of a child?

- A study is performed where 18 children with congenital heart malformation are followed for observation of when the children first were able to walk.
  The mean age in months was 14.1

Congenital heart malformation and motor development – test II

- From large studies of normal children it has been shown that the mean age of children at their first steps alone is 13 months with a standard deviation of 1.75 months.

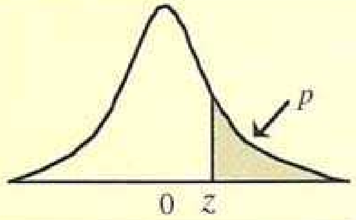
- Based on the problem in question a null-hypothesis (H_0) is defined:
  - Children with congenital heart malformation has the same mean age when they learn to walk
  - H_0: $\mu = 13$ months

Congenital heart malformation and motor development – test III

- Assume a normal distribution of the mean and assume that the H_0 is true, then:

- Calculate the probability that a random sample of 18 children has a mean “as far away from” 13 as 14.1. This is the p-value.

- If the p-value is small (less than 0.05), this means that what we have observed is unlikely to observe. Self-contradiction. Reject the underlying assumption.



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
	Second decimal place of z									
0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641
0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
0.7	.2420	.2389	.2358	.2327	.2297	.2266	.2236	.2206	.2177	.2148
0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026

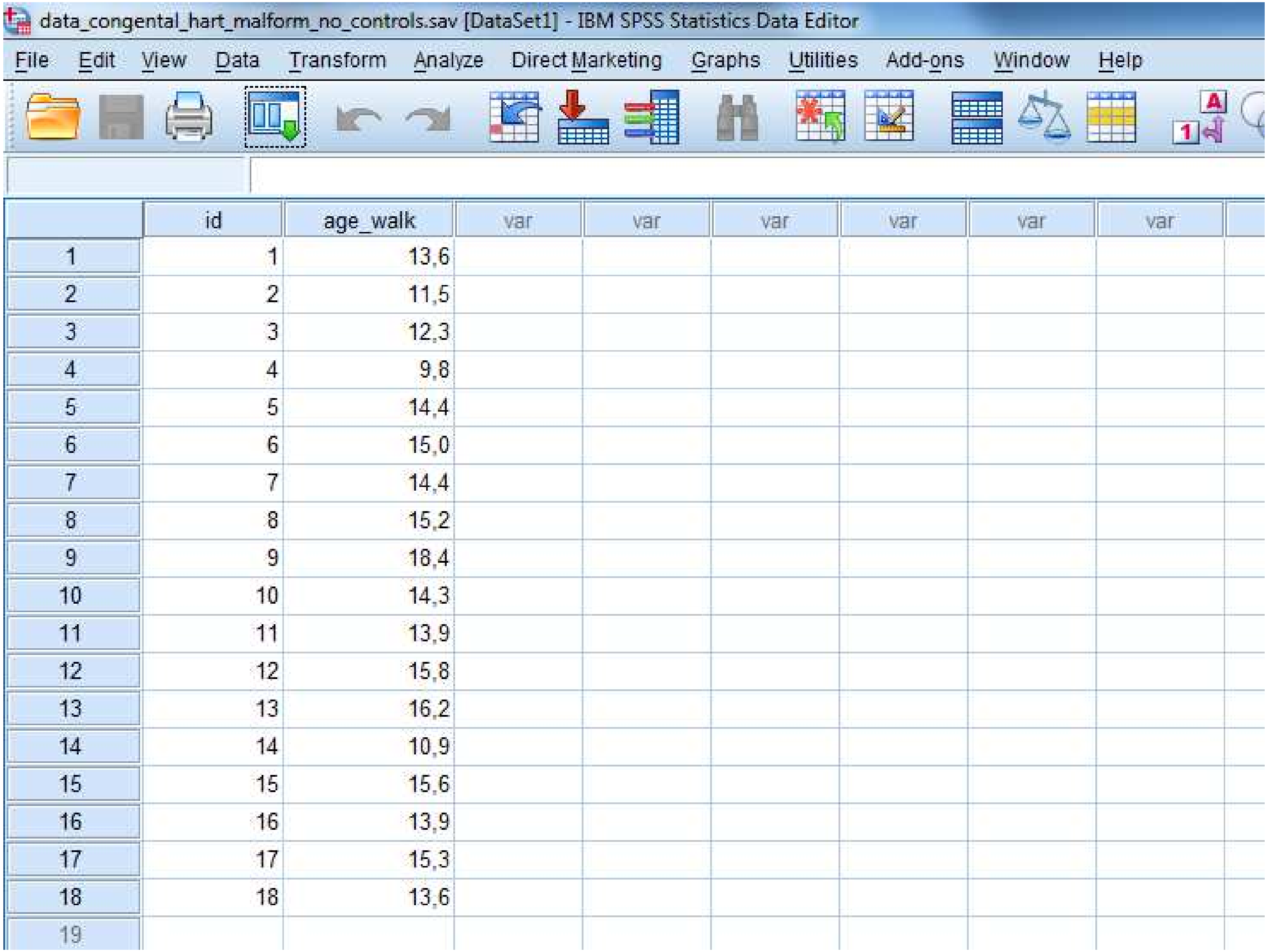
t-test

- When the standard error cannot be assumed known, we will use the standard error in the sample (SD) as an estimate of the standard deviation of the population (σ).
- Instead of a z-value, we will compute a t-value for calculating the p-value.
- $t = (\bar{X} - \mu_0)/(SD/\sqrt{n})$
- P-value is found in a t-table using (n-1) degrees of freedom

Table A3 Percentage points of the *t* distribution.

Adapted from Table 7 of White *et al.* (1979) with permission of the authors and publishers.

	One-sided *P*-value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided *P*-value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29



	id	age_walk	var	var	var	var	var	var
1	1	13,6						
2	2	11,5						
3	3	12,3						
4	4	9,8						
5	5	14,4						
6	6	15,0						
7	7	14,4						
8	8	15,2						
9	9	18,4						
10	10	14,3						
11	11	13,9						
12	12	15,8						
13	13	16,2						
14	14	10,9						
15	15	15,6						
16	16	13,9						
17	17	15,3						
18	18	13,6						
19								

T-Test

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One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
age_walk	18	14.120	2.0391	.4806

One-Sample Test

	Test Value = 13					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
age_walk	2.331	17	.032	1.1202	.106	2.134

T-Test

```
[DataSet1] O:\TEXT\UNDERV\HELSTA\data_congental_hart_malform_no_controls.sav
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One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
age_walk	18	14,120	2,0391	,4806

One-Sample Test

	Test Value = 13					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
age_walk	2,331	17	,032	1,1202	,106	2,134

	id	age_walk	group	var	var
1	1	13,6	1		
2	2	11,5	1		
3	3	12,3	1		
4	4	9,8	1		
5	5	14,4	1		
6	6	15,0	1		
7	7	14,4	1		
8	8	15,2	1		
9	9	18,4	1		
10	10	14,3	1		
11	11	13,9	1		
12	12	15,8	1		
13	13	16,2	1		
14	14	10,9	1		
15	15	15,6	1		
16	16	13,9	1		
17	17	15,3	1		
18	18	13,6	1		
19	19	12,8	2		
20	20	13,6	2		
21	21	15,8	2		
22	22	13,2	2		
23	23	10,7	2		
24	24	12,8	2		
25	25	12,2	2		
26	26	11,8	2		
27	27	10,7	2		
28	28	14,0	2		
29	29	12,0	2		
30	30	12,8	2		
31	31	12,7	2		
32	32	16,3	2		
33	33	13,6	2		
34	34	11,5	2		
35	35	13,4	2		
36	36	12,4	2		
37	37	11,8	2		
38	38	14,2	2		
39					

Two sample t-test

$H_0: \mu_a = \mu_b$

Comparing the means between two groups A and B

$$\frac{\bar{X}_a - \bar{X}_b}{SEM(\bar{X}_a - \bar{X}_b)\sqrt{1/n_a + 1/n_b}}$$

$$= t_{n_a+n_b-2}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{s^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \qquad s^2 = \frac{\sum_{j=1}^{n_1}(x_j - \bar{x}_1)^2 + \sum_{i=1}^{n_2}(x_i - \bar{x}_2)^2}{n_1 + n_2 - 2}$$

T-Test

```
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Group Statistics

	group	N	Mean	Std. Deviation	Std. Error Mean
age_walk	Con_hart	18	14,120	2,0391	,4806
	Control	20	12,920	1,4497	,3242

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
age_walk	Equal variances assumed	1,219	,277	2,108	36	,042	1,2003	,5695	,0453	2,3553
	Equal variances not assumed			2,070	30,362	,047	1,2003	,5797	,0169	2,3836

Another example of test

- Problem: To what extent is the mental health influenced by being next of kin to a person with cancer?
- A study is measuring mental health (by SF-36) of 30 persons being next of kin to a person with cancer.

9. These questions are about how you feel and how things have been with you during the past 4 weeks. For each question, please give the one answer that comes closest to the way you have been feeling. How much of the time during the past 4 weeks.

a. did you feel full of pep?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

b. have you been a very nervous person?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

c. have you felt so down in the dumps nothing could cheer you up?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

d. have you felt calm and peaceful?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

e. did you have a lot of energy?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

f. have you felt downhearted and blue?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

g. did you feel worn out?

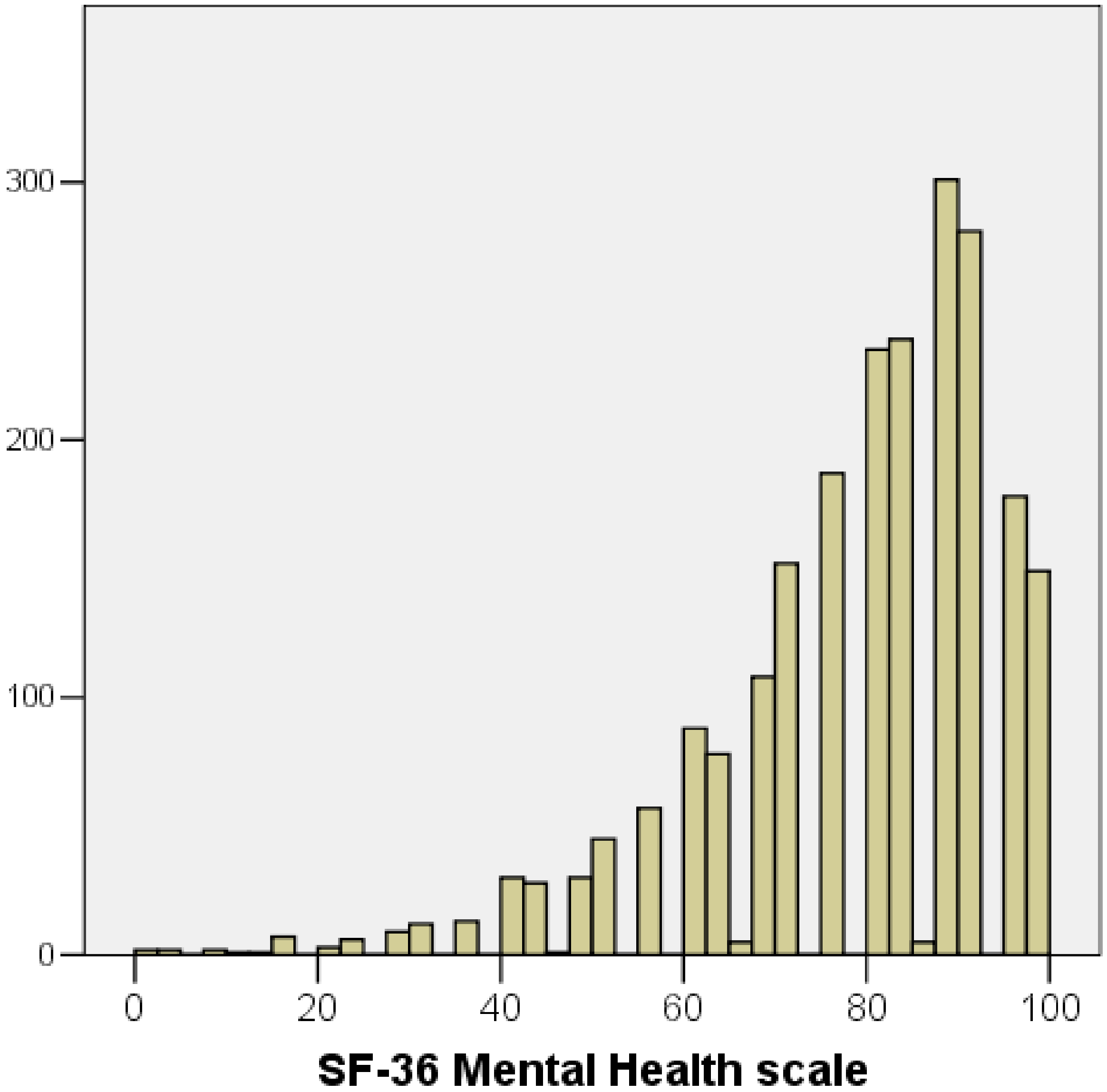
- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

h. have you been a happy person?

- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time

i. did you feel tired?

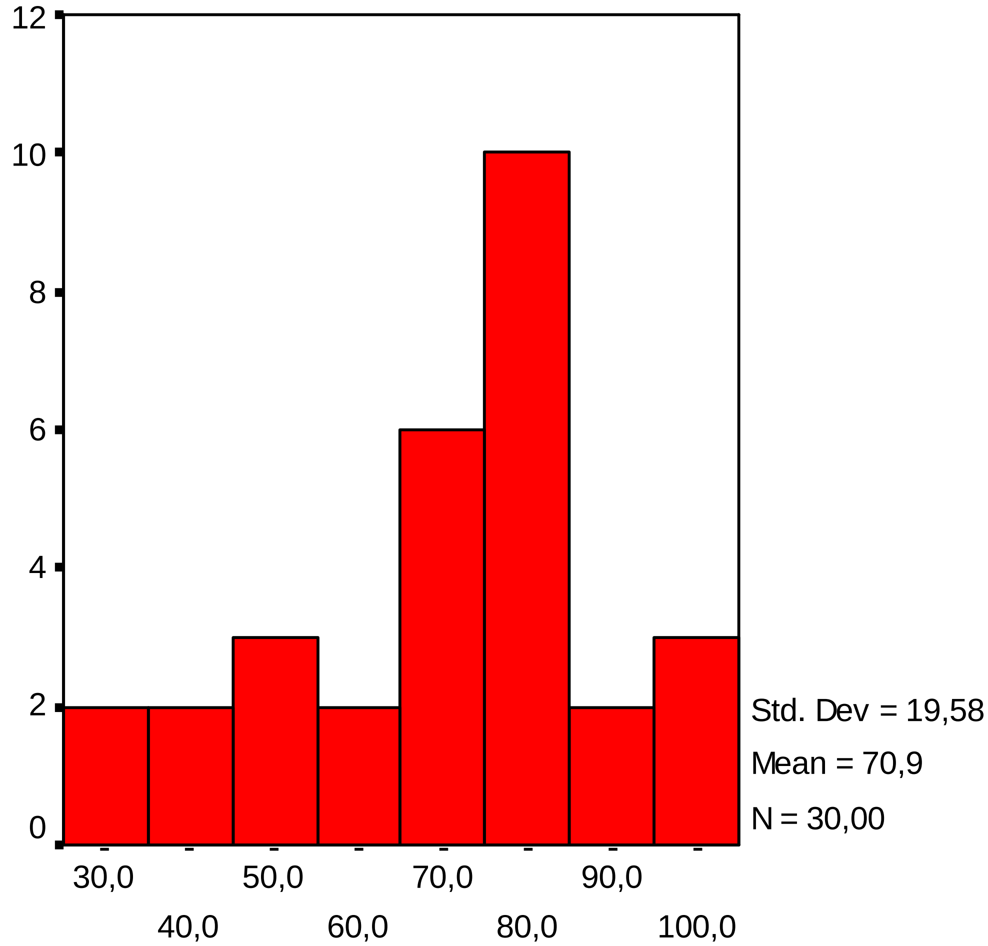
- ❑ All of the time
- ❑ Most of the time
- ❑ A good bit of the time
- ❑ Some of the time
- ❑ A little of the time
- ❑ None of the time



Mean = 78.7

SD = 15.4

N = 2255



SF-36 Mental Health

Statistical test -1

- Problem:
  - How is the mental health (measured by SF-36) of persons next of kin to cancer patients?
- Define the nullhypothesis based on the problem:
  - H_0: Next of kin to cancer patients have the same level of mental health as the general population(μ_0 = 79).
- Calculated the mean of the sample og 30 persons next of kin
  - $\overline{X} = 71$

Statistisk test - 2

- Calculated the p-value (probabilty of obtaining a mean that is as far from 79 as the one we got – here using SD from the general population)

$2 \cdot P(\overline{X} \leq 71) =$

$2 \cdot P\left((\overline{X} - 79) / \frac{15}{\sqrt{30}} \leq (71-79) / \frac{15}{\sqrt{30}}\right) =$

* $2 \cdot P(Z \leq -2.92) = 2 \cdot P(Z \geq 2.92) = 2 \cdot 0.0017 = 0.0034$

Generally: $2 \cdot P(Z \geq (\overline{X} - \mu_0) / \frac{\sigma}{\sqrt{n}})$

* Using the assumption that $\overline{X}$ is normally distributed with mean 79 and SD = $\frac{15}{\sqrt{30}}$

Statistical test - 2

- Calculated the p-value (probabilty of obtaining a mean that is as far from 79 as the one we got – here using SD from the general population)

  $2 \cdot P(\overline{X} \leq 71) =$

  $2 \cdot P\left((\overline{X} - 79) / \frac{15}{\sqrt{30}} \leq (71-79) / \frac{15}{\sqrt{30}}\right) =$

  $2 \cdot P(Z \leq -2.92) = 2 \cdot P(Z \geq 2.92) = 2 \cdot 0.0017 = 0.0034$

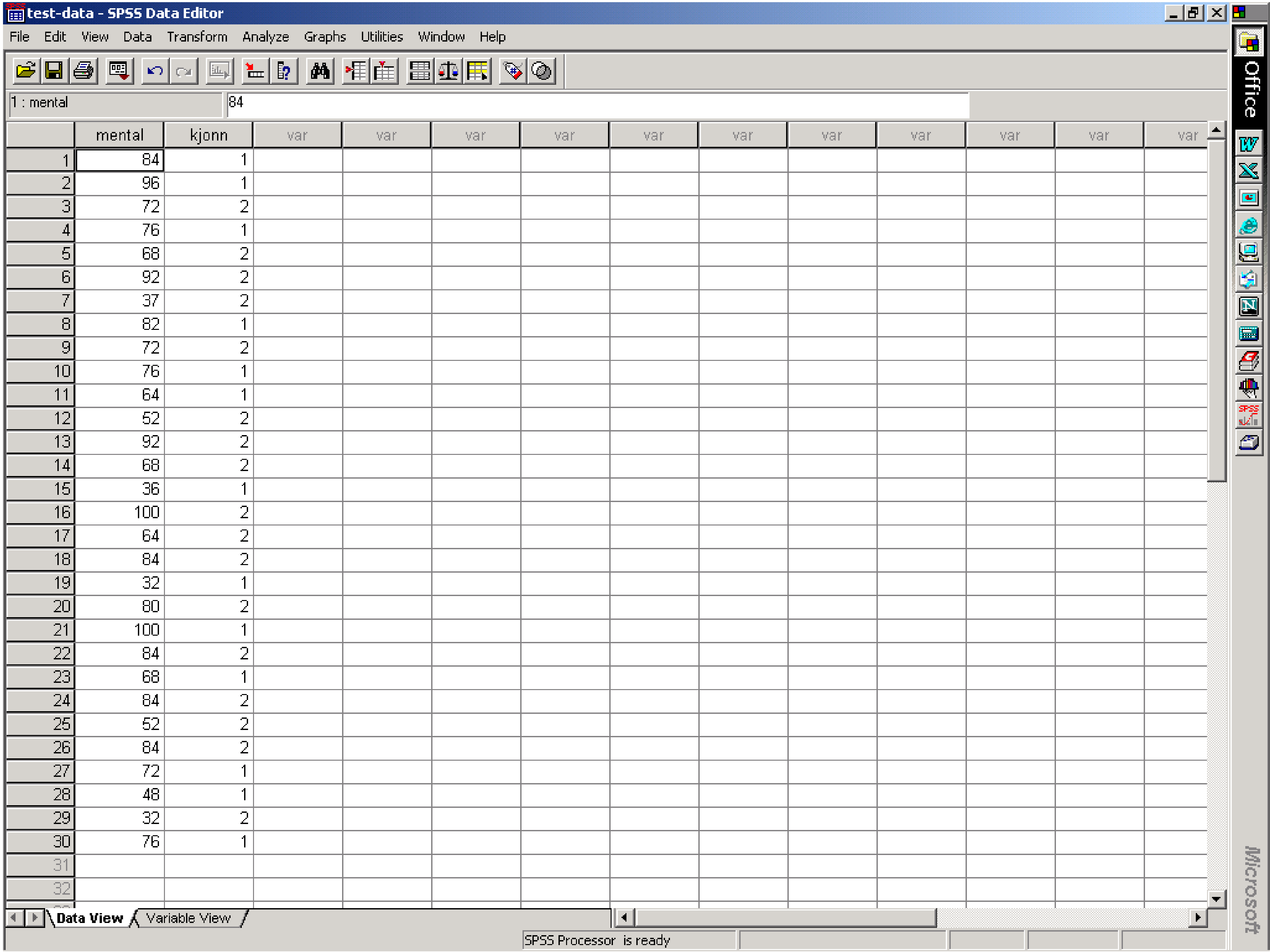
Statistical test - 3

- If the p-value is less than the significance level α (i.e. 0.05) the H_0 is rejected.

- If the p-value is greater that α then H_0 may not be rejected. This does NOT mean that we have confirmed H_0, only that these data cannot be used to reject H_0.

t-test

- When the standard deviation cannot be assumed known, we will use the standard deviation in the sample (SD) as an estimate of the standard deviation of the population (σ).
- Instead of a z-value, we will compute a t-value for calculating the p-value.
- $t = (\bar{X} - \mu_0)/(SD/\sqrt{n})$
- P-value is found in a t-table using (n-1) degrees of freedom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7 37 2
8 82 1
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10 76 1
11 64 1
12 52 2
13 92 2
14 68 2
15 36 1
16 100 2
17 64 2
18 84 2
19 32 1
20 80 2
21 100 1
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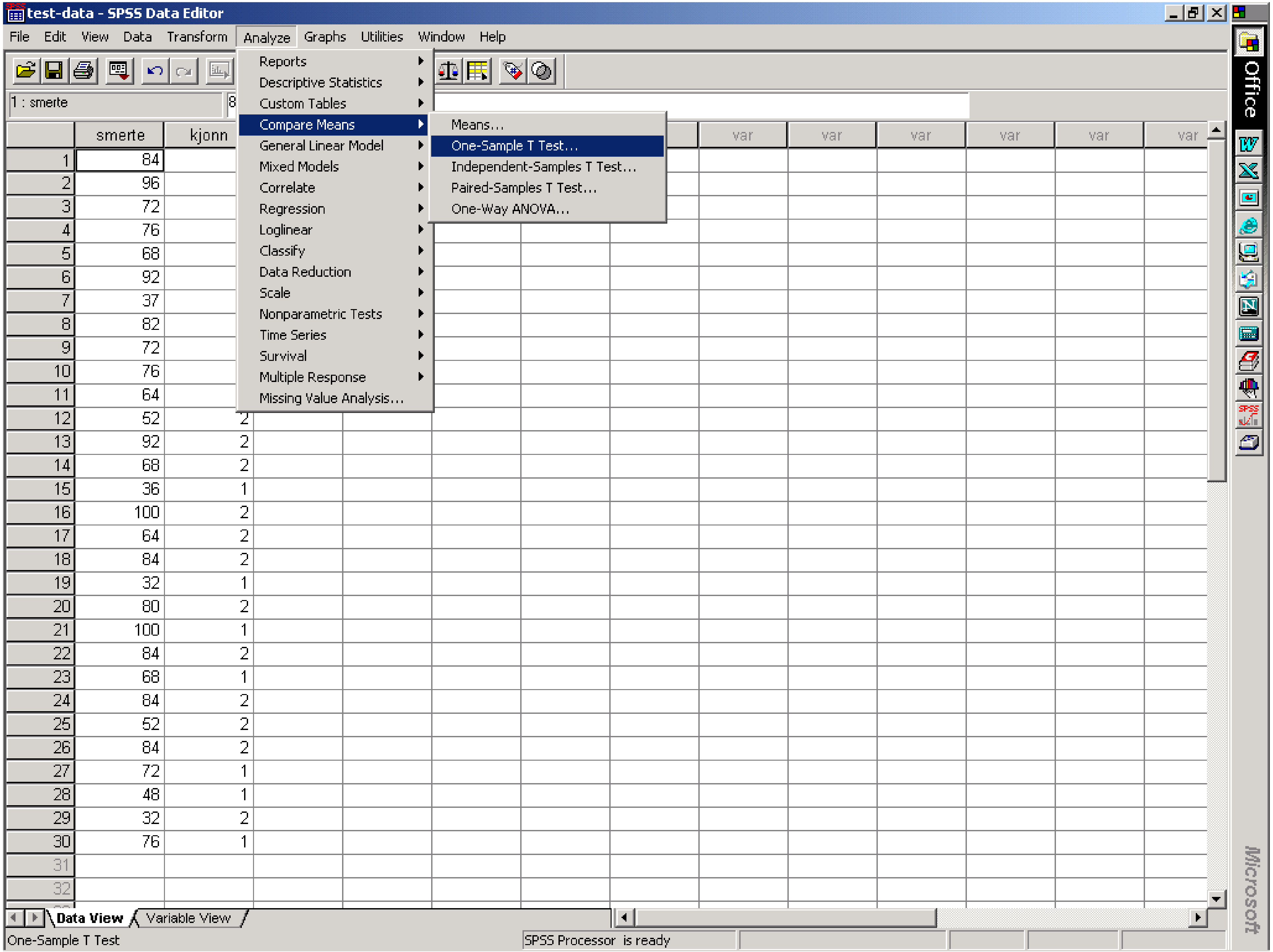

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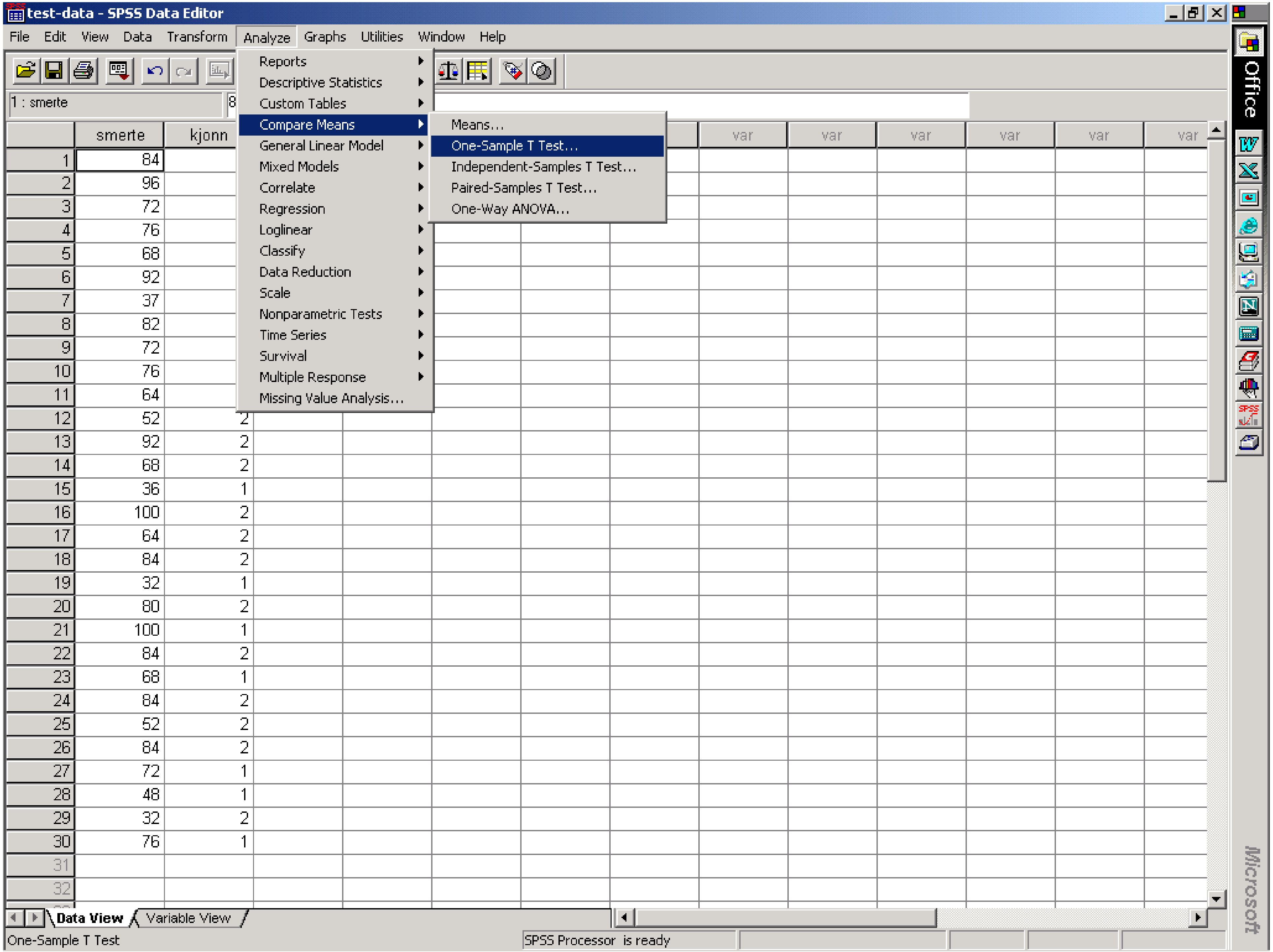
t-test

- When the standard deviation cannot be assumed known, we will use the standard deviation in the sample (SD) as an estimate of the standard deviation of the population (σ).
- Instead of a z-value, we will compute a t-value for calculating the p-value.
- $t = (\bar{X} - \mu_0)/(SD/\sqrt{n})$
- P-value is found in a t-table using (n-1) degrees of freedom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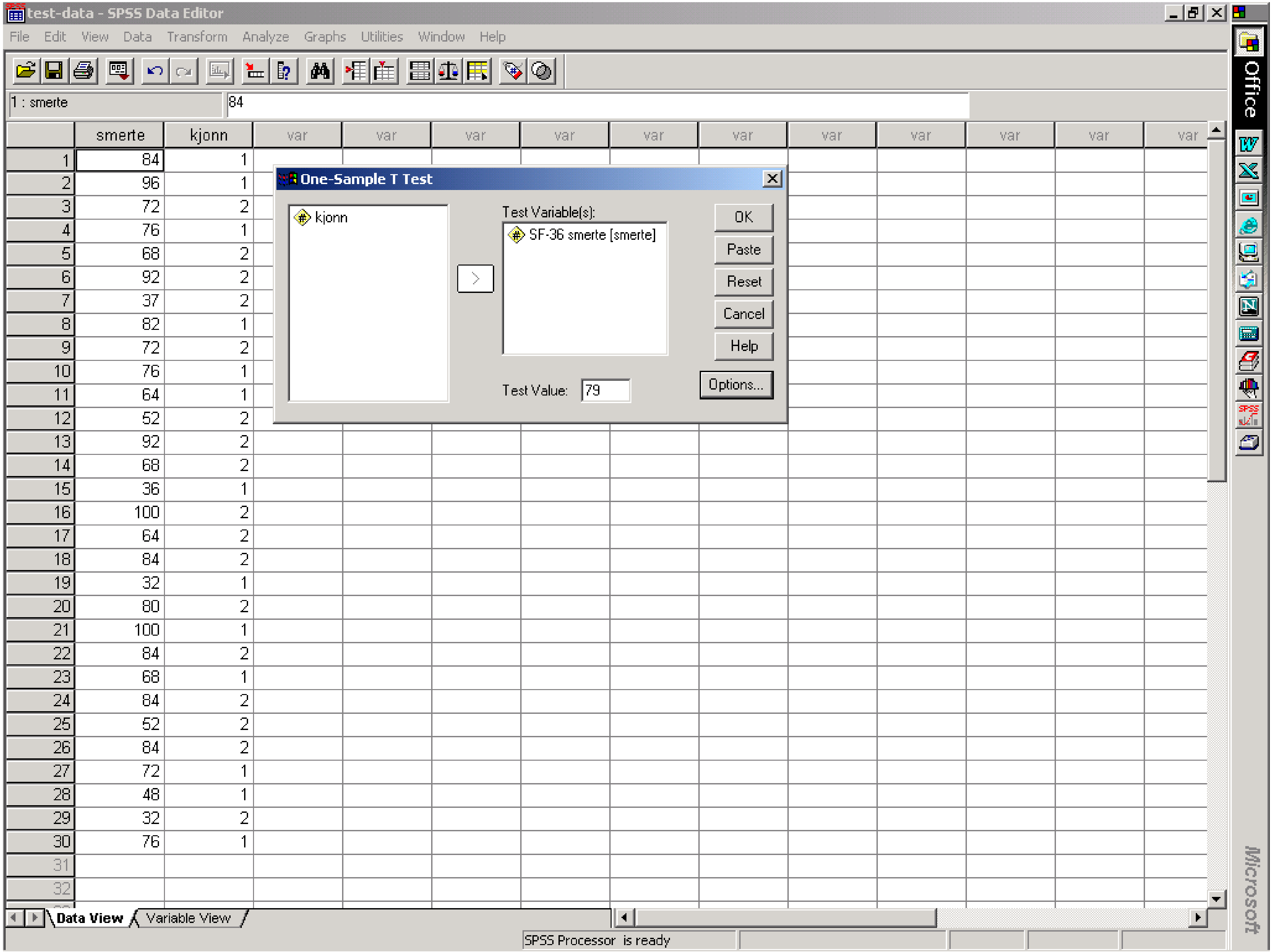
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Test Value: 79
OK
Paste
Reset
Cancel
Help
Options...
1 84 1
2 96 1
3 72 2
4 76 1
5 68 2
6 92 2
7 37 2
8 82 1
9 72 2
10 76 1
11 64 1
12 52 2
13 92 2
14 68 2
15 36 1
16 100 2
17 64 2
18 84 2
19 32 1
20 80 2
21 100 1
22 84 2
23 68 1
24 84 2
25 52 2
26 84 2
27 72 1
28 48 1
29 32 2
30 76 1
Data View
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SPSS Processor is ready
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T-Test

One-Sample Statistics

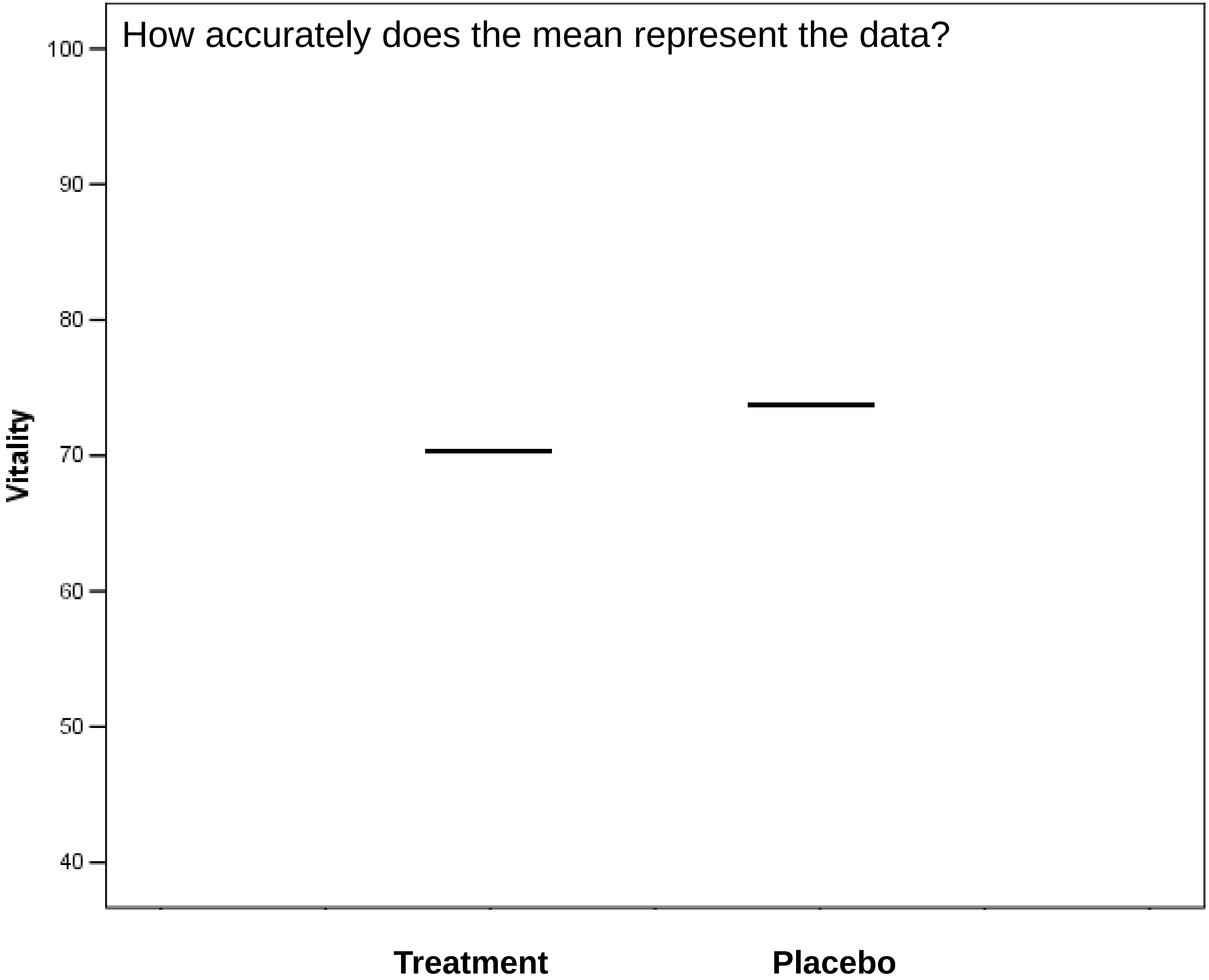
	N	Mean	Std. Deviation	Std. Error Mean
SF-36 mental helse	30	70,90	19,575	3,574

One-Sample Test

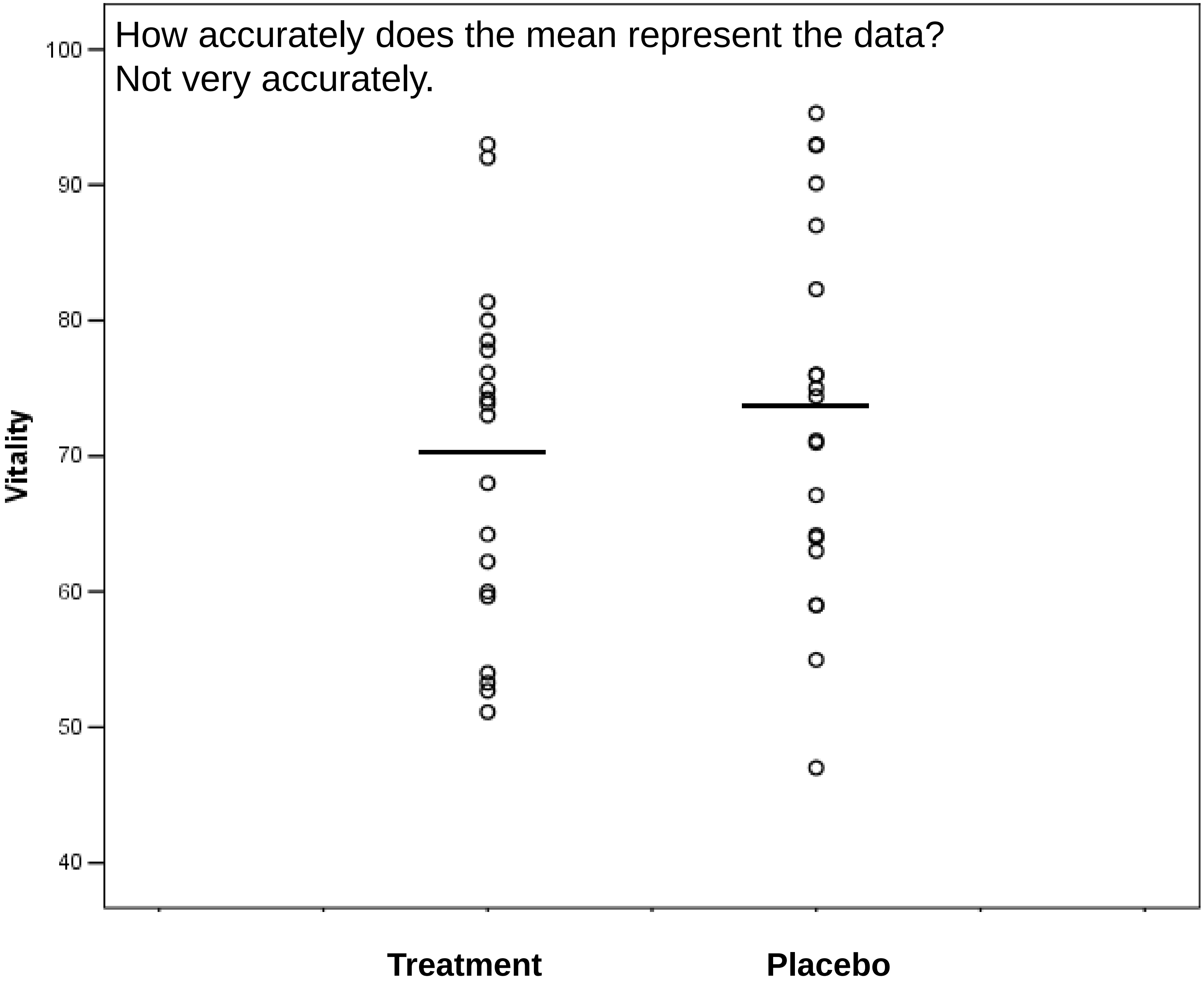
	Test Value = 79					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
SF-36 mental helse	-2,266	29	,031	-8,10	-15,41	-,79

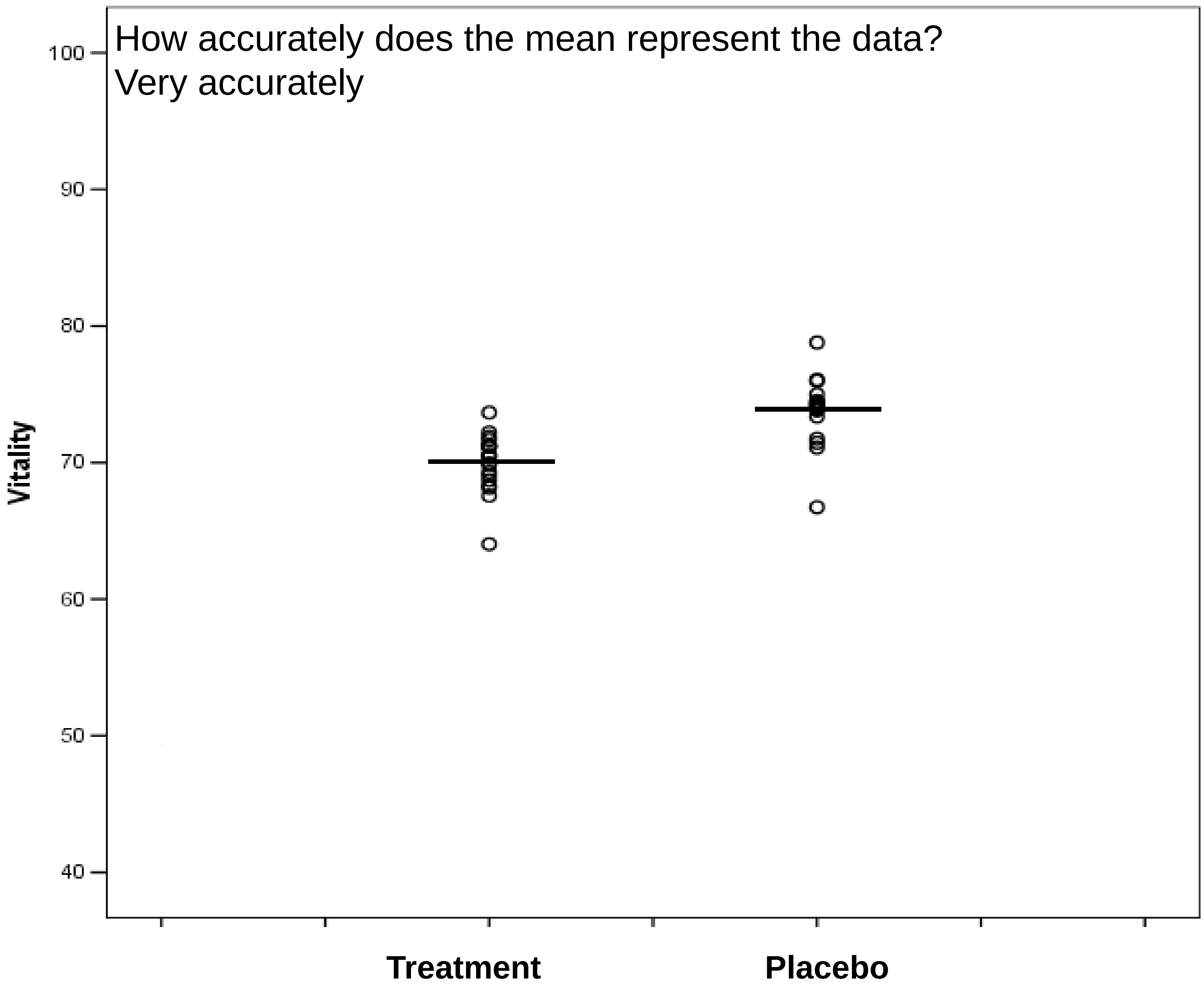
Paired t-test

- Paired samples *t*-tests are used when we have a sample of matched pairs of similar units, or one group of units that has been tested twice (a "repeated measures" *t*-test).
- $X_{diff} = X_{after} - X_{before}$
- We use a one sample t-test on X_{diff} with (n-1) d.f.
- $t = (\bar{X} - \mu_0)/(SD/\sqrt{n})$


How accurately does the mean represent the data?
100
90
80
70
60
50
40
Vitality
Treatment
Placebo

How accurately does the mean represent the data?
Not very accurately.




How accurately does the mean represent the data?
Very accurately
Vitality
100
90
80
70
60
50
40
Treatment
Placebo

Confidence interval

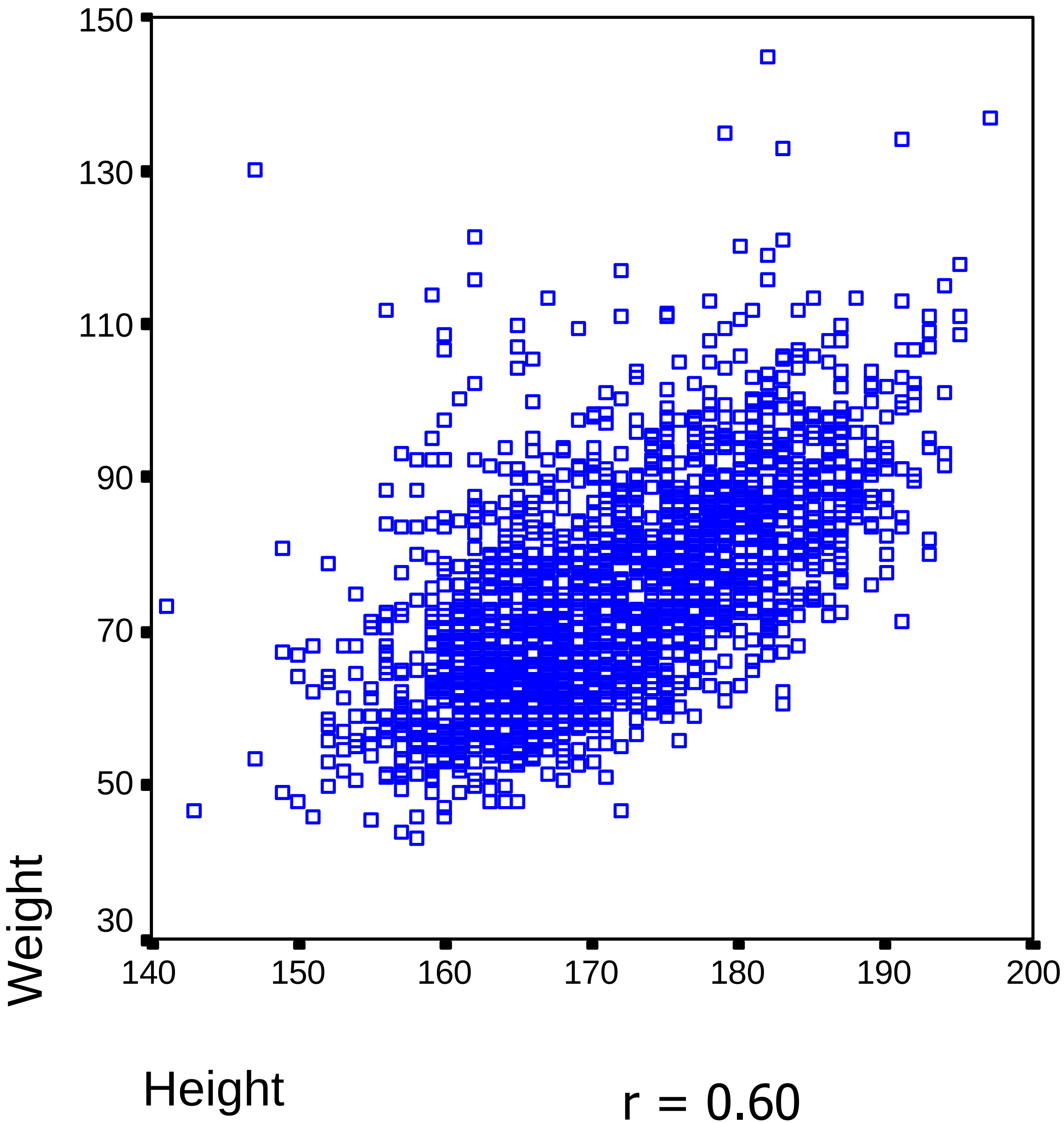
- The confidence interval (CI) is providing an interval where the «true» mean in the in population where the sample was drawn from is included with a certain probability (i.e. 95%).
- Using the information on the distribution of the mean the CI can be calculated as:
- 95% CI: $[\ \bar{X} \pm t_{0.975} \cdot \text{SEM}\ ]$

  $= [\ \bar{X} \pm t_{0.975} \cdot \text{SD}/\sqrt{n}\ ]$

Table A3 Percentage points of the *t* distribution.

Adapted from Table 7 of White *et al.* (1979) with permission of the authors and publishers.

	One-sided *P*-value								
	0.25	0.1	0.05	0.025	0.01	0.005	0.0025	0.001	0.0005
	Two-sided *P*-value								
d.f.	0.5	0.2	0.1	0.05	0.02	0.01	0.005	0.002	0.001
1	1.00	3.08	6.31	12.71	31.82	63.66	127.32	318.31	636.62
2	0.82	1.89	2.92	4.30	6.96	9.92	14.09	22.33	31.60
3	0.76	1.64	2.35	3.18	4.54	5.84	7.45	10.21	12.92
4	0.74	1.53	2.13	2.78	3.75	4.60	5.60	7.17	8.61
5	0.73	1.48	2.02	2.57	3.36	4.03	4.77	5.89	6.87
6	0.72	1.44	1.94	2.45	3.14	3.71	4.32	5.21	5.96
7	0.71	1.42	1.90	2.36	3.00	3.50	4.03	4.78	5.41
8	0.71	1.40	1.86	2.31	2.90	3.36	3.83	4.50	5.04
9	0.70	1.38	1.83	2.26	2.82	3.25	3.69	4.30	4.78
10	0.70	1.37	1.81	2.23	2.76	3.17	3.58	4.14	4.59
11	0.70	1.36	1.80	2.20	2.72	3.11	3.50	4.02	4.44
12	0.70	1.36	1.78	2.18	2.68	3.06	3.43	3.93	4.32
13	0.69	1.35	1.77	2.16	2.65	3.01	3.37	3.85	4.22
14	0.69	1.34	1.76	2.14	2.62	2.98	3.33	3.79	4.14
15	0.69	1.34	1.75	2.13	2.60	2.95	3.29	3.73	4.07
16	0.69	1.34	1.75	2.12	2.58	2.92	3.25	3.69	4.02
17	0.69	1.33	1.74	2.11	2.57	2.90	3.22	3.65	3.96
18	0.69	1.33	1.73	2.10	2.55	2.88	3.20	3.61	3.92
19	0.69	1.33	1.73	2.09	2.54	2.86	3.17	3.58	3.88
20	0.69	1.32	1.72	2.09	2.53	2.84	3.15	3.55	3.85
21	0.69	1.32	1.72	2.08	2.52	2.83	3.14	3.53	3.82
22	0.69	1.32	1.72	2.07	2.51	2.82	3.12	3.50	3.79
23	0.68	1.32	1.71	2.07	2.50	2.81	3.10	3.48	3.77
24	0.68	1.32	1.71	2.06	2.49	2.80	3.09	3.47	3.74
25	0.68	1.32	1.71	2.06	2.48	2.79	3.08	3.45	3.72
26	0.68	1.32	1.71	2.06	2.48	2.78	3.07	3.44	3.71
27	0.68	1.31	1.70	2.05	2.47	2.77	3.06	3.42	3.69
28	0.68	1.31	1.70	2.05	2.47	2.76	3.05	3.41	3.67
29	0.68	1.31	1.70	2.04	2.46	2.76	3.04	3.40	3.66
30	0.68	1.31	1.70	2.04	2.46	2.75	3.03	3.38	3.65
40	0.68	1.30	1.68	2.02	2.42	2.70	2.97	3.31	3.55
60	0.68	1.30	1.67	2.00	2.39	2.66	2.92	3.23	3.46
120	0.68	1.29	1.66	1.98	2.36	2.62	2.86	3.16	3.37
∞	0.67	1.28	1.65	1.96	2.33	2.58	2.81	3.09	3.29


150
130
110
90
70
50
30
140
150
160
170
180
190
200
Weight
Height

r = 0.60

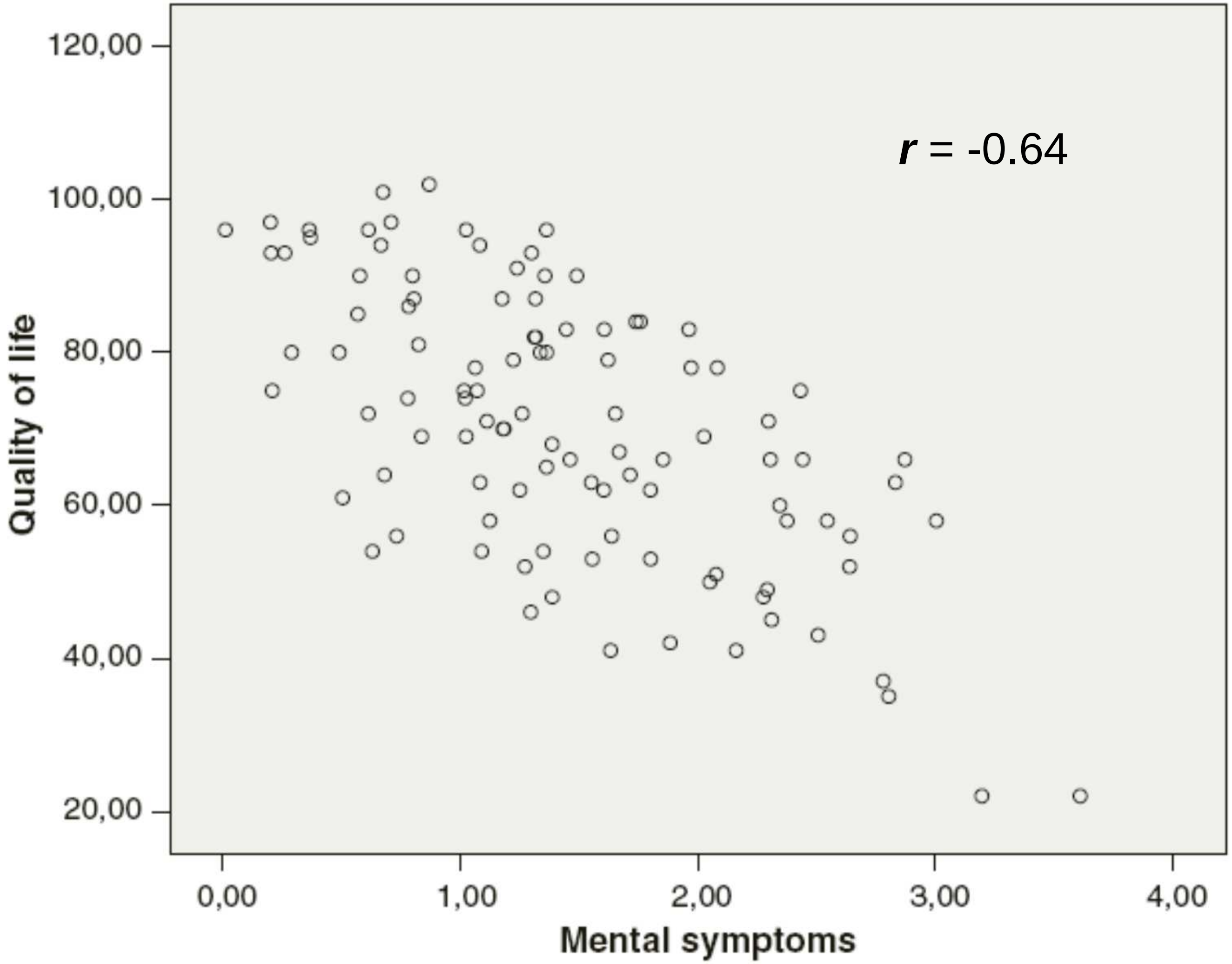
Correlation

- The correlation coefficient (***r***) measures the strength and direction of the linear association between two ordinal variables
- It varies between -1 and 1;
  - r = 0 indicates no association
  - r = 1 indicates a perfect positive association
  - r = -1 indicates a perfect negative association

Formula for calculating the correlation coefficient

$$r = \frac{\sum_{i=1}^{n}\left(\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)\right)}{\sqrt{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2 \sum_{i=1}^{n}\left(y_i - \bar{y}\right)^2}}$$

Correlation Between Mental Symptoms and Quality of Life in the MHP Group

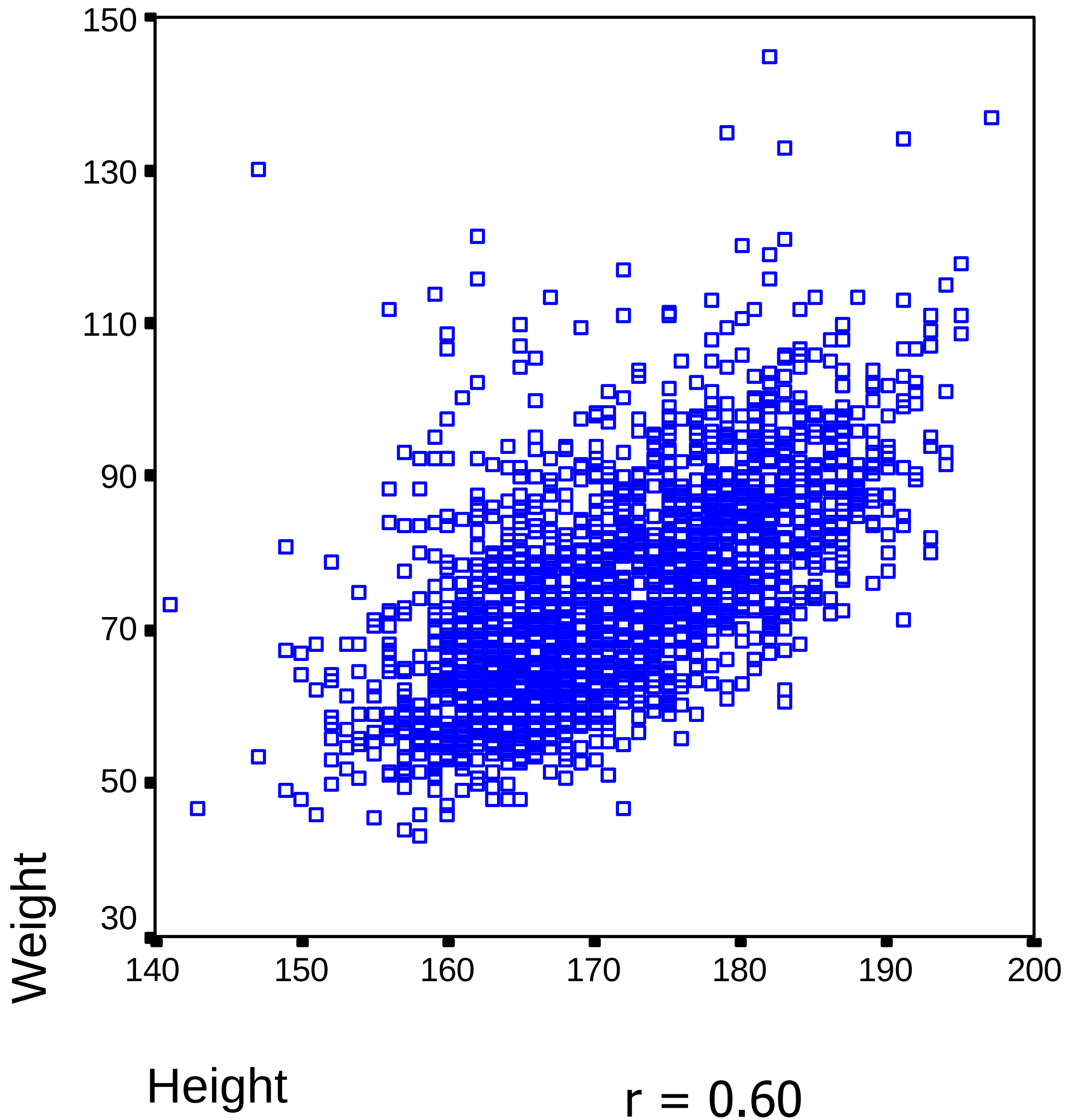


Langeland et al. Community Ment Health J 2007;43:321-39

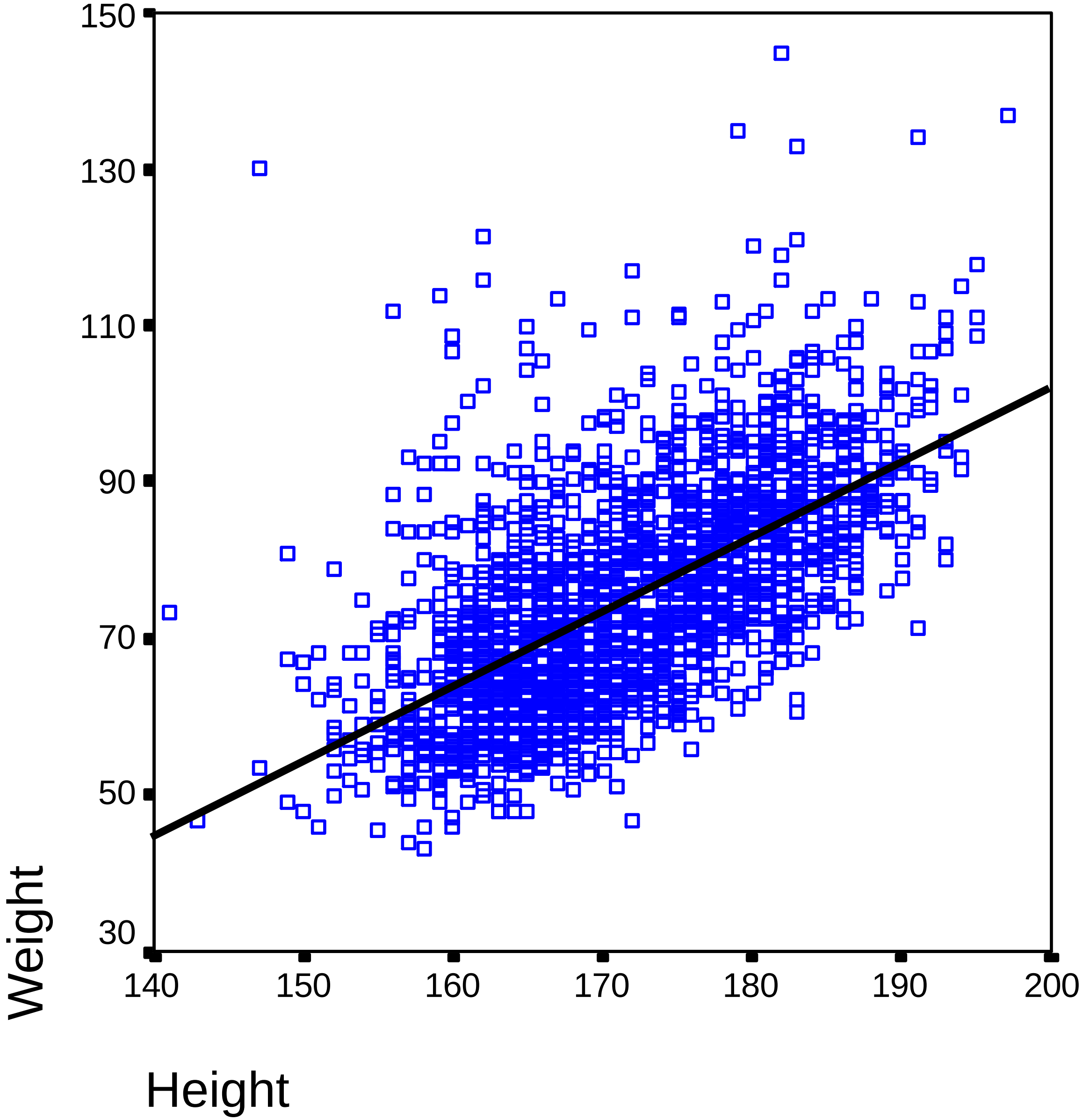
Regression analysis

- A regression analysis is modelling an association between one (or more) response variables (dependent variables) and predictor variables (independent variable).
- The association is estimated as regression coefficients.
- The regression coefficient indicates how much the dependent variable is changing when the independent variable is changing one unit.

$$\mathbf{Y = \beta_0 + \beta_1 x_{1,i} + \ldots + \beta_k x_{k,i}}$$


150
130
110
90
70
50
30
Weight
140
150
160
170
180
190
200
Height

r = 0.60



r = 0.60

Weight = -97 + 0.94 · Height

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,599[a]	,358	,358	10,9939

a. Predictors: (Constant), hoyde

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-86,178	6,876		-12,534	,000
	hoyde	,939	,040	,599	23,539	,000

a. Dependent Variable: vektkg

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
vektkg	994	41,0	142,0	75,458	13,7182
lettmosj	970	1	4	3,25	,828
hardmosj	965	1	4	2,37	1,007
hoyde	994	147	197	172,13	8,745
ald1997	1000	40	47	43,13	2,139
Valid N (listwise)	940				

Correlations

		vektkg	hoyde	lettmosj	hardmosj	ald1997
vektkg	Pearson Correlation	1	,599**	-,058	-,008	-,029
	Sig. (2-tailed)		,000	,073	,808	,359
	N	994	994	966	960	994
hoyde	Pearson Correlation	,599**	1	-,015	,091**	-,062*
	Sig. (2-tailed)	,000		,641	,005	,050
	N	994	994	966	960	994
lettmosj	Pearson Correlation	-,058	-,015	1	,417**	,030
	Sig. (2-tailed)	,073	,641		,000	,344
	N	966	966	970	944	970
hardmosj	Pearson Correlation	-,008	,091**	,417**	1	,015
	Sig. (2-tailed)	,808	,005	,000		,650
	N	960	960	944	965	965
ald1997	Pearson Correlation	-,029	-,062*	,030	,015	1
	Sig. (2-tailed)	,359	,050	,344	,650	
	N	994	994	970	965	1000

**. Correlation is significant at the 0.01 level (2-tailed).

*. Correlation is significant at the 0.05 level (2-tailed).

Group Statistics

	kjona	N	Mean	Std. Deviation	Std. Error Mean
vektkg	Mann	477	82,696	11,5064	,5268
	Kvinne	517	68,779	12,1199	,5330

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
vektkg	Equal variances assumed	,035	,851	18,531	992	,000	13,9171	,7510	12,4434	15,3909
	Equal variances not assumed			18,570	991,186	,000	13,9171	,7495	12,4464	15,3878

Model Summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	,613[a]	,376	,373	10,9163

a. Predictors: (Constant), hardmosj, kjona, lettmosj, hoyde

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	67133,350	4	16783,338	140,841	,000[a]
	Residual	111419,3	935	119,165		
	Total	178552,6	939			

a. Predictors: (Constant), hardmosj, kjona, lettmosj, hoyde

b. Dependent Variable: vektkg

Coefficients[a]

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	-50,730	11,379		-4,458	,000
	hoyde	,785	,059	,496	13,250	,000
	kjona	-4,214	1,036	-,153	-4,067	,000
	lettmosj	-,202	,479	-,012	-,422	,673
	hardmosj	-,811	,395	-,059	-2,051	,041

a. Dependent Variable: vektkg

Estimation of expected value in a regression analysis

Weight = – 50.7 + 0.79 height – 4.2 sex – 0.20 lightex – 0.81 heavex

Expl. Woman of 160 cm, light exercise 1-2 times a week and heavy exercise 1-2 times a week

Weight = – 50.7 + 0.79 x 160 – 4.2 x 2 – 0.20 x 3 – 0.81 x 3

Weight = – 50.7 + 126.4 – 8.4 – 0.60 – 2.24 = 64.5

A regression analysis is modeling the level of a linear association between variables

Cannot estimate non-linear associations

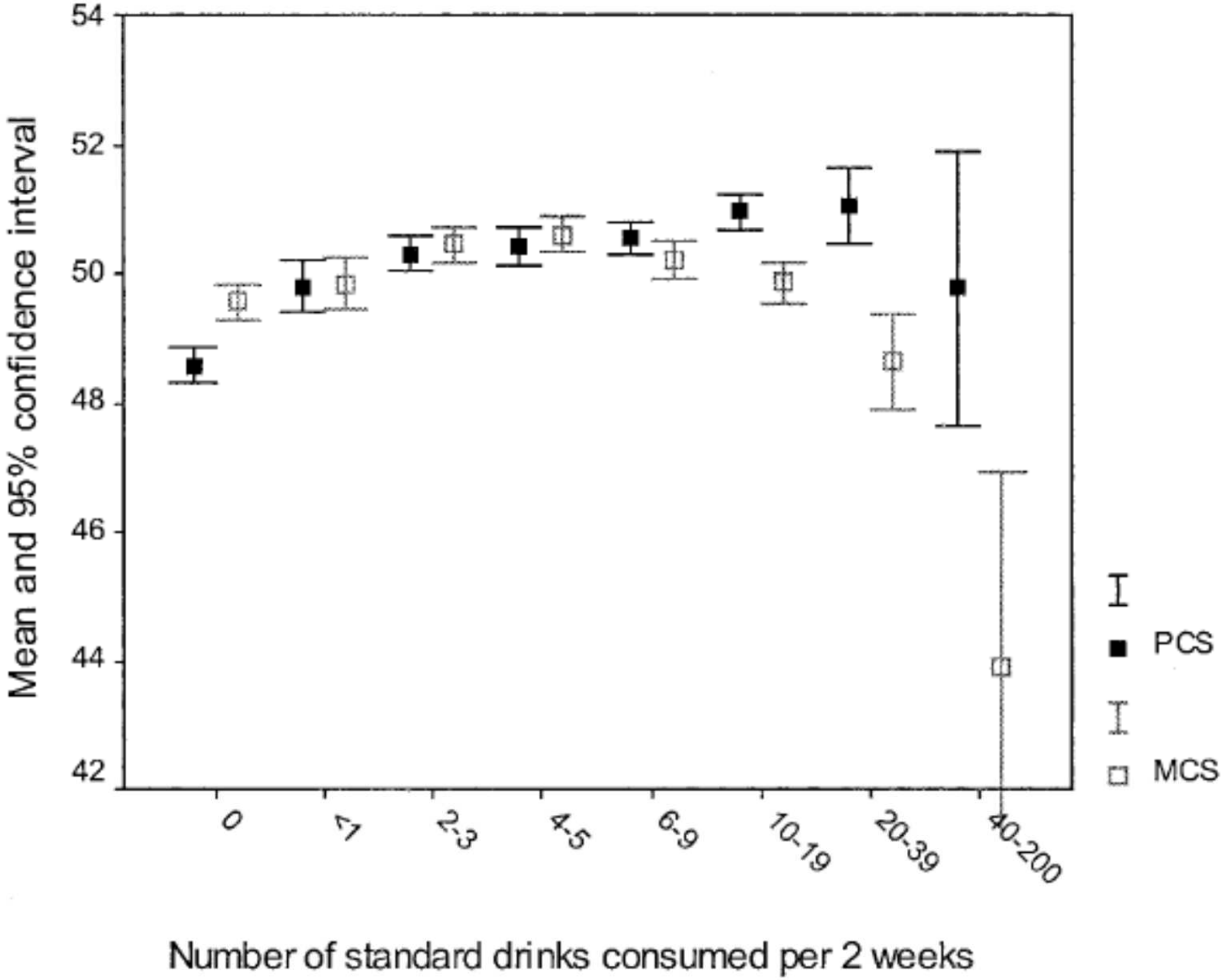
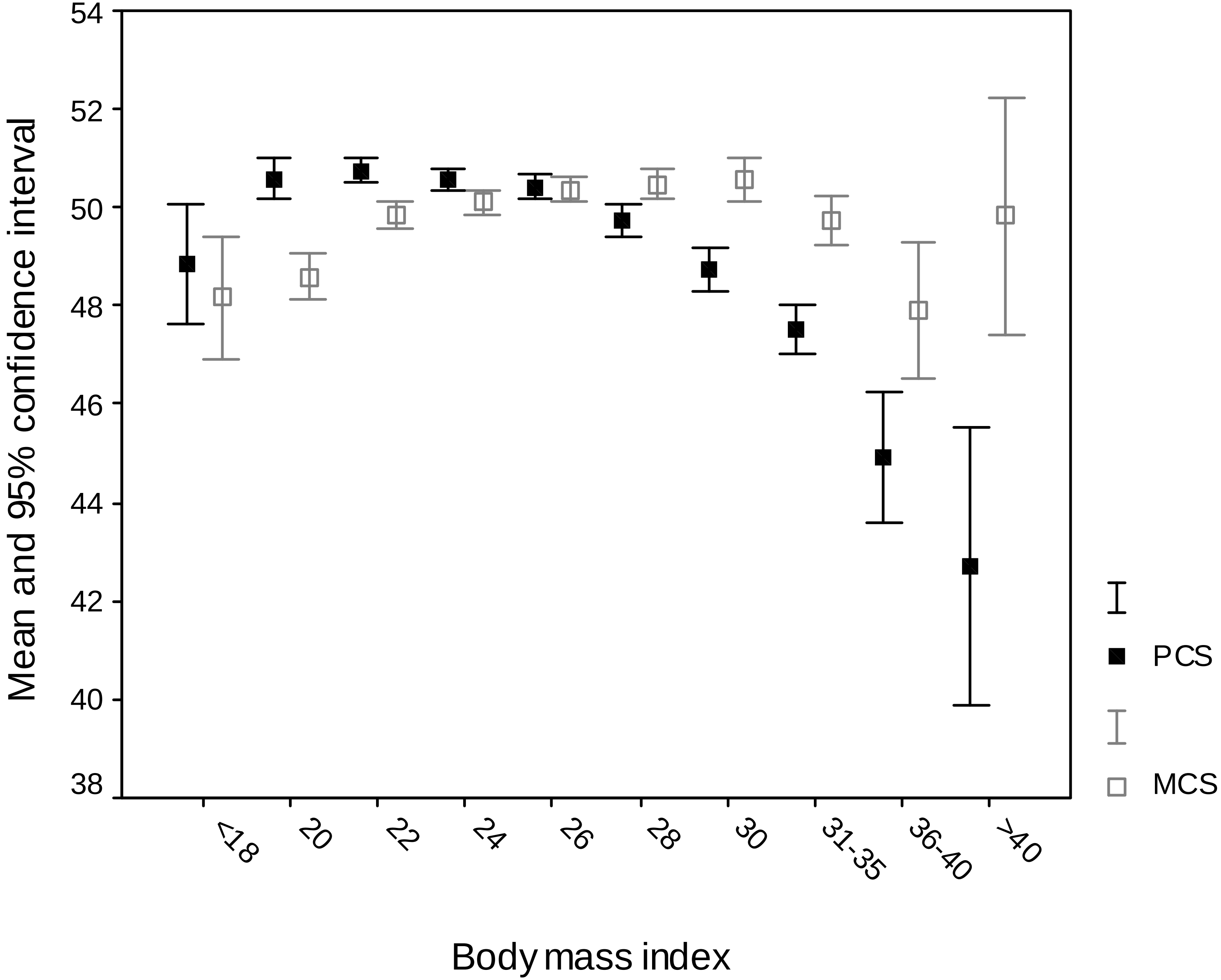


Fig. 1. Alcohol consumption and health-related quality of life (SF-12 Health Survey) among 22,312 subjects in Hordaland County, Norway aged 40 – 47 years.



Mean and 95% confidence interval
54
52
50
48
46
44
42
40
38
<18
20
22
24
26
28
30
31-35
36-40
>40
Body mass index
PCS
MCS

A statistically significant association between two variables does not neccesarily mean that there is a causal relationship between the two variables

Oldenburg

Chi-square test (χ^2-test)

- To test the association between two variables on a nominal level.
- Observed number (O) for each combination of values of the two variables is listed in a table.
- Expected number (E) for each cell is calculated.
- $\chi^2 = \Sigma(O - E)^2/E$ has a chi-square distribution with $(r-1)\cdot(s-1)$ degrees of freedom, where r and s are the number of catgories for each of the two variables

Chi-square test - example

Pain during labor – is there a difference in the perceived level of pain between women giving birth for the first time and women who have given birth before?

Level of pain

	Weak or moderate pain	Strong or unbearable pain	
First time mothers	**A** E = (A+B)(A+C)/n	**B** E = (A+B)(B+D)/n	A+B
Given birth before	**C** E = (A+C)(C+D)/n	**D** E = (B+D)(C+D)/n	C+D
	A+C	B+D	n = A+B+C+D

Chi-square test - example

Pain during labor – is there a difference in the perceived level of pain between women giving birth for the first time and women who have given birth before?

Level of pain

	Weak or moderate pain	Strong or unbearable pain	
First time mothers	**33** 45%	**40** 55%	73 100%
Given birth before	**46** 62%	**28** 38%	74 100%
	79	68	147

Relative risk (RR) = proportion of women with strong pain among first time mothers - divided by the proportion of women with strong pain among women having given birth before

RR = 55% / 38% = 1.45

Chi-square test - example

Level of pain

	Weak or moderate pain	Strong or unbearable pain	
First time mothers	**33** E = 39.2	**40** E = 33.8	73
Given birth before	**46** E = 39.8	**28** E = 34.2	74
	79	68	147

$\chi^2 = \Sigma(O - E)^2/E = (33\text{-}39)^2/39.2 + (40\text{-}33.8)^2/33.8 + ... = 4.25$

One degree of freedom: p-value between 0.05 and 0.025 (table A5).

Table A5 Percentage points of the χ^2 distribution.

	P-value							
d.f.	0.5	0.25	0.1	0.05	0.025	0.01	0.005	0.001
1	0.45	1.32	2.71	3.84	5.02	6.63	7.88	10.83
2	1.39	2.77	4.61	5.99	7.38	9.21	10.60	13.82
3	2.37	4.11	6.25	7.81	9.35	11.34	12.84	16.27
4	3.36	5.39	7.78	9.49	11.14	13.28	14.86	18.47
5	4.35	6.63	9.24	11.07	12.83	15.09	16.75	20.52
6	5.35	7.84	10.64	12.59	14.45	16.81	18.55	22.46
7	6.35	9.04	12.02	14.07	16.01	18.48	20.28	24.32
8	7.34	10.22	13.36	15.51	17.53	20.09	21.96	26.13
9	8.34	11.39	14.68	16.92	19.02	21.67	23.59	27.88
10	9.34	12.55	15.99	18.31	20.48	23.21	25.19	29.59
11	10.34	13.70	17.28	19.68	21.92	24.73	26.76	31.26
12	11.34	14.85	18.55	21.03	23.34	26.22	28.30	32.91
13	12.34	15.98	19.81	22.36	24.74	27.69	29.82	34.53
14	13.34	17.12	21.06	23.68	26.12	29.14	31.32	36.12
15	14.34	18.25	22.31	25.00	27.49	30.58	32.80	37.70
16	15.34	19.37	23.54	26.30	28.85	32.00	34.27	39.25
17	16.34	20.49	24.77	27.59	30.19	33.41	35.72	40.79
18	17.34	21.60	25.99	28.87	31.53	34.81	37.16	42.31
19	18.34	22.72	27.20	30.14	32.85	36.19	38.58	43.82
20	19.34	23.83	28.41	31.41	34.17	37.57	40.00	45.32
21	20.34	24.93	29.62	32.67	35.48	38.93	41.40	46.80
22	21.34	26.04	30.81	33.92	36.78	40.29	42.80	48.27
23	22.34	27.14	32.01	35.17	38.08	41.64	44.18	49.73
24	23.34	28.24	33.20	36.42	39.36	42.98	45.56	51.18
25	24.34	29.34	34.38	37.65	40.65	44.31	46.93	52.62
26	25.34	30.43	35.56	38.89	41.92	45.64	48.29	54.05
27	26.34	31.53	36.74	40.11	43.19	46.96	49.64	55.48
28	27.34	32.62	37.92	41.34	44.46	48.28	50.99	56.89
29	28.34	33.71	39.09	42.56	45.72	49.59	52.34	58.30
30	29.34	34.80	40.26	43.77	46.98	50.89	53.67	59.70
40	39.34	45.62	51.81	55.76	59.34	63.69	66.77	73.40
50	49.33	56.33	63.17	67.50	71.42	76.15	79.49	86.66
60	59.33	66.98	74.40	79.08	83.30	88.38	91.95	99.61
70	69.33	77.58	85.53	90.53	95.02	100.43	104.22	112.32
80	79.33	88.13	96.58	101.88	106.63	112.33	116.32	124.84
90	89.33	98.65	107.57	113.15	118.14	124.12	128.30	137.21
100	99.33	109.14	118.50	124.34	129.56	135.81	140.17	149.45

røyk * hjertesykdom Crosstabulation

			hjertesykdom Ja	hjertesykdom Nei	Total
røyk	Ja	Count	205	13958	14163
		% within røyk	1,4%	98,6%	100,0%
	Nei	Count	58	8096	8154
		% within røyk	,7%	99,3%	100,0%
Total		Count	263	22054	22317
		% within røyk	1,2%	98,8%	100,0%

H0: No difference in risk of heart disease between smokers and non-smokers

H1: Difference in risk

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)	Exact Sig. (2-sided)	Exact Sig. (1-sided)
Pearson Chi-Square	24,078[b]	1	,000		
Continuity Correction[a]	23,450	1	,000		
Fisher's Exact Test				,000	,000
N of Valid Cases	22317				

a. Computed only for a 2x2 table

b. 0 cells (,0%) have expected count less than 5. The minimum expected count is 96,09.

H0 rejected because $p<0.05$

Risk Estimate

	Value	95% Confidence Interval Lower	95% Confidence Interval Upper	
Odds Ratio for røyk (Nei / Ja)	2,050	1,530	2,747	OR
For cohort hjertesykdom = Ja	2,035	1,522	2,720	RR
N of Valid Cases	22317			

H0 rejected because 95% confidence interval did not contain 1

kjona * hjertesykdom Crosstabulation

			hjertesykdom		Total
			Ja	Nei	
kjona	Mann	Count	169	10094	10263
		% within kjona	1,6%	98,4%	100,0%
	Kvinne	Count	94	11960	12054
		% within kjona	,8%	99,2%	100,0%
Total		Count	263	22054	22317
		% within kjona	1,2%	98,8%	100,0%

H0: No difference in risk of heart disease between males and females

H1: Difference in risk

Risk Estimate

	Value	95% Confidence Interval		
		Lower	Upper	
Odds Ratio for kjona (Mann / Kvinne)	2,130	1,653	2,745	OR
For cohort hjertesykdom = Ja	2,112	1,643	2,714	RR
N of Valid Cases	22317			

H0 rejected?

kjona * hjertesykdom Crosstabulation

			hjertesykdom		Total
			Ja	Nei	
kjona	Mann	Count	169	10094	10263
		% within kjona	1,6%	98,4%	100,0%
	Kvinne	Count	94	11960	12054
		% within kjona	,8%	99,2%	100,0%
Total		Count	263	22054	22317
		% within kjona	1,2%	98,8%	100,0%

H0: No difference in risk of heart disease between males and females

H1: Difference in risk

Risk Estimate

	Value	95% Confidence Interval		
		Lower	Upper	
Odds Ratio for kjona (Mann / Kvinne)	2,130	1,653	2,745	OR
For cohort hjertesykdom = Ja	2,112	1,643	2,714	RR
N of Valid Cases	22317			

H0 rejected? Yes, because 1 is not contained in the 95% confidence interval(s)

ald1997 * hjertesykdom Crosstabulation

			hjertesykdom		
			Ja	Nei	Total
ald1997	40	Count	26	3636	3662
		% within ald1997	,7%	99,3%	100,0%
	41	Count	36	3692	3728
		% within ald1997	1,0%	99,0%	100,0%
	42	Count	38	3703	3741
		% within ald1997	1,0%	99,0%	100,0%
	43	Count	52	3692	3744
		% within ald1997	1,4%	98,6%	100,0%
	44	Count	45	3661	3706
		% within ald1997	1,2%	98,8%	100,0%
	46	Count	35	1753	1788
		% within ald1997	2,0%	98,0%	100,0%
	47	Count	31	1917	1948
		% within ald1997	1,6%	98,4%	100,0%
Total		Count	263	22054	22317
		% within ald1997	1,2%	98,8%	100,0%

H0: Age does not matter
H1: Age matters

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	22,835[a]	6	,001
N of Valid Cases	22317		

a. 0 cells (,0%) have expected count less than 5. The minimum expected count is 21,07.

H0 rejected?

ald1997 * hjertesykdom Crosstabulation

			hjertesykdom		
			Ja	Nei	Total
ald1997	40	Count	26	3636	3662
		% within ald1997	,7%	99,3%	100,0%
	41	Count	36	3692	3728
		% within ald1997	1,0%	99,0%	100,0%
	42	Count	38	3703	3741
		% within ald1997	1,0%	99,0%	100,0%
	43	Count	52	3692	3744
		% within ald1997	1,4%	98,6%	100,0%
	44	Count	45	3661	3706
		% within ald1997	1,2%	98,8%	100,0%
	46	Count	35	1753	1788
		% within ald1997	2,0%	98,0%	100,0%
	47	Count	31	1917	1948
		% within ald1997	1,6%	98,4%	100,0%
Total		Count	263	22054	22317
		% within ald1997	1,2%	98,8%	100,0%

H0: Age does not matter

H1: Age matters

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	22,835[a]	6	,001
N of Valid Cases	22317		

a. 0 cells (,0%) have expected count less than 5. The minimum expected count is 21,07.

H0 rejected? Yes, because $p<0.05$

alccat2 * hjertesykdom Crosstabulation

			hjertesykdom		Total
			Ja	Nei	
alccat2	1	Count	65	4968	5033
		% within alccat2	1,3%	98,7%	100,0%
	2	Count	22	1932	1954
		% within alccat2	1,1%	98,9%	100,0%
	3	Count	34	3110	3144
		% within alccat2	1,1%	98,9%	100,0%
	4	Count	24	2853	2877
		% within alccat2	,8%	99,2%	100,0%
	5	Count	39	3159	3198
		% within alccat2	1,2%	98,8%	100,0%
	6	Count	35	2774	2809
		% within alccat2	1,2%	98,8%	100,0%
	7	Count	13	619	632
		% within alccat2	2,1%	97,9%	100,0%
	8	Count	1	61	62
		% within alccat2	1,6%	98,4%	100,0%
Total		Count	233	19476	19709
		% within alccat2	1,2%	98,8%	100,0%

H0: Alcohol use does not matter

H1: Alcohol use matters

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	8,197[a]	7	,316
N of Valid Cases	19709		

a. 1 cells (6,3%) have expected count less than 5. The minimum expected count is ,73.

H0 rejected?

alccat2 * hjertesykdom Crosstabulation

			hjertesykdom		Total
			Ja	Nei	
alccat2	1	Count	65	4968	5033
		% within alccat2	1,3%	98,7%	100,0%
	2	Count	22	1932	1954
		% within alccat2	1,1%	98,9%	100,0%
	3	Count	34	3110	3144
		% within alccat2	1,1%	98,9%	100,0%
	4	Count	24	2853	2877
		% within alccat2	,8%	99,2%	100,0%
	5	Count	39	3159	3198
		% within alccat2	1,2%	98,8%	100,0%
	6	Count	35	2774	2809
		% within alccat2	1,2%	98,8%	100,0%
	7	Count	13	619	632
		% within alccat2	2,1%	97,9%	100,0%
	8	Count	1	61	62
		% within alccat2	1,6%	98,4%	100,0%
Total		Count	233	19476	19709
		% within alccat2	1,2%	98,8%	100,0%

H0: Alcohol use does not matter

H1: Alcohol use matters

Chi-Square Tests

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	8,197[a]	7	,316
N of Valid Cases	19709		

a. 1 cells (6,3%) have expected count less than 5. The minimum expected count is ,73.

H0 rejected? No, because $p>0.05$